Appendix A

The basic equations are:

$$\frac{dX_1}{dt} = -\alpha X_1 X_2 \tag{A1}$$

$$\frac{dX_2}{dt} = \alpha X_1 X_2 - \gamma X_2 \tag{A2}$$

The boundary conditions at $t = 0$ in the data are:

$$X_1(0) = (N - a) \text{ and } X_2(0) = a \tag{A3}$$

Given these boundary conditions, $X_1(t)$ and $X_2(t)$ are related by a modified version of Eq. (12):

$$X_2(t) = N - a + \frac{N}{R}\log\left(\frac{X_1(t)}{N-a}\right) - X_1(t) \tag{A4}$$

For small t, we can expand the log in (A4) as follows:

$$\log\left(\frac{X_1(t)}{N-a}\right) = \log\left[1 - \left(1 - \left(\frac{X_1(t)}{N-a}\right)\right)\right] \sim -\left(1 - \left(\frac{X_1(t)}{N-a}\right)\right) \tag{A5}$$

which, when substituted into (A4) gives, after some simple algebra,

$$X_2(t) = \frac{R-1}{R}\left[N - X_1(t)\left(1 - \frac{a}{(R-1)N}\right)\right] \sim \frac{R-1}{R}\left[N - X_1(t)\right] \tag{A6}$$

Substituting these into (A1), we get a Logistic Equation:

$$\frac{dX_1}{dt} = -\gamma(R-1)X_1(t)\left[1 - \frac{X_1(t)}{N}\right] \tag{A7}$$

whose solution, with the boundary condition (A3) is:

$$X_1(t) = \frac{N}{\left[1 + \frac{a}{N-a}e^{\gamma(R-1)t}\right]} \tag{A8}$$

For small t, we can expand the denominator to get:

$$X_1(t) \sim N\left[1 - \frac{a}{N-a}e^{\gamma(R-1)t}\right], \text{ t small} \tag{A9}$$

Substituting into (A6) gives,

$$X_2(t) \sim \frac{(R-1)aN}{R(N-a)}e^{\gamma(R-1)t} \tag{A10}$$

This analysis shows that for small t, $X_1(t)$ decreases exponentially from its initial value of $(N - a)$ and this decrease feeds an exponential increase in $X_2(t)$.

(A10 shows that the coefficient of t in the exponent of $X_2(t)$ is $\gamma(R - 1)$. (A11)