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Overestimate of Committed Warming: Supplementary Information

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A simple coupled ice sheet-temperature-carbon model

We describe a three-component model for glacial cycles that includes two-way coupling between ice sheets and temperature, and between temperature and carbon dioxide levels. This is perhaps the simplest model that allows us to distinguish correlations between temperature and CO_2 in the presence of high latitude oscillatory forcing of the ice sheets resulting from shifts in the Earth's orbit from the Earth System Sensitivity (ESS), the long-term response (including ice sheets) of the system to CO_2 forcing.

We define three prognostic variables as means and anomalies: T_0 and $T(t)$ for temperature (K); C_0 and $C(t)$ for CO_2 (ppm); L_0 and $L(t)$ for sea level (meters of sea level equivalent, mSL). (Anomalous ice amount is the negative of L). For mid-glacial conditions, we take T_0, C_0 and L_0 as 285 K, 230 ppm and -60 mSL, respectively. The governing equations of this simple model are as follows:

$$\tau_L \frac{dL}{dt} = F_I + (aT - L) \tag{1}$$

$$\tau_C \frac{dC}{dt} = bT - C \tag{2}$$

$$H \frac{dT}{dt} = \mu L + F_{CO_2}(C) + \lambda T - P(T, T_0) + \epsilon \tag{3}$$

where in Eq. (1) F_I is the high latitude forcing of the ice sheets, a is the sensitivity of ice sheets to temperature (mSL K^{-1}) and τ_L is a time constant for the response of ice sheets. In Eq. (2), b is the sensitivity of the carbon cycle to temperature (ppm K^{-1}) and τ_C is the carbon cycle response time. In Eq. (3), H is the heat capacity of the system (but this falls out if one assumes that T is in balance with C and L over these timescales); μ is the radiative forcing associated with the ice sheet (W m^{-2}) mSL^{-1}, λ is the non-Planck feedback W m^{-2} K^{-1}, $P(T, T_0)$ is the Planck function, and ϵ is a source of random noise, N(0,0.2) W m^{-2}. Eq. (3) includes the standard radiative forcing for carbon dioxide $F_{CO_2}(C) = 5.3 \log((C + C_0)/C_0)$ and the full fast-feedback term (split into the Planck feedback and the non-Planck terms). The Planck function in its full form is $P(T, T_0) = \sigma((T + T_0)^4 - T_0^4)$ (where σ is the Stefan-Boltzmann constant). The parameters of this model are summarized in Table 1.

If we consider longer-than-millennial variations, we can assume that temperature is in equilibrium with the ice and CO_2, thus Eq. (3) is a polynomial in T (that can have zero to 2 solutions depending on the parameters and the form of the Planck function). The equations are stepped forward with time steps of 1000 years. To prevent unphysical values, we force CO_2 to remain positive (minimum value is 1 ppm, corresponding to a radiative forcing of -29 W m^{-2} with respect to pre-industrial levels).

The Charney sensitivity (K for $2\times CO_2$) in this system can be calculated by setting $C = C_0$ and fixing $L = 0$. The Earth System Sensitivity additionally allows for a change in the ice in response to increased CO_2 and is calculated similarly assuming L is in equilibrium with T with $F == 0$, i.e. $L = aT$. The regression between T and the radiative forcing associated with C over the glacial cycles is also easily calculated, and when scaled by multiplying by 3.7 W m^{-2} has the same units as the ESS. Note that in situations where the model becomes unstable, the regression is only calculated over a partial time-series.

Some parameters can be estimated roughly based on inferences from the real world[1] (outlined in Table S1). For the simulations below we specifically choose μ=0.025 W m^{-2} mSL^{-1}, b=20 ppm K^{-1}, τ_L is 10000 yrs and τ_C is 3000 years. The high latitude forcing in the specific case looked at here is $F = 30 \sin(\pi t/\tau_F)$, where τ_F is 40,000 yrs, giving a 60 mSL peak-to-trough driving function and ≈12 glacial cycles over a million simulated years. We performed 20×20 experiments varying λ (so that we vary the Charney sensitivity roughly within the accepted range), and varying a between 0 and 30 mSL K^{-1}.

With the full Planck response, the Charney sensitivity and ESS don't have a simple analytical expression, but with the range of λ of [2.8,4.6] W m^{-2} K^{-1} and a of [0,30]mSL K^{-1}, give rise to a range of $S_{2\times CO2}$ of [1.5,5.6]K and ESS of [1.5,>30]K.

In the main text we show two example time series of T, C and L for (λ,a) = (3.2,6.2) and (4.1,22.1), which are equivalent to a Charney Sensitivity/ESS pair of (1.7,1.9)K and (3.0,5.1)K respectively (Main Text Fig. 1a). The scaled regression between T and the radiative forcing due to C in each case is near 7.8K (Fig. S1).

Over the whole range of the experiments the ratio of regression (after scaling for $2\times CO_2$) and the actual model ESS is shown in Main text Fig. 1b. If the regression was a good estimate for ESS,

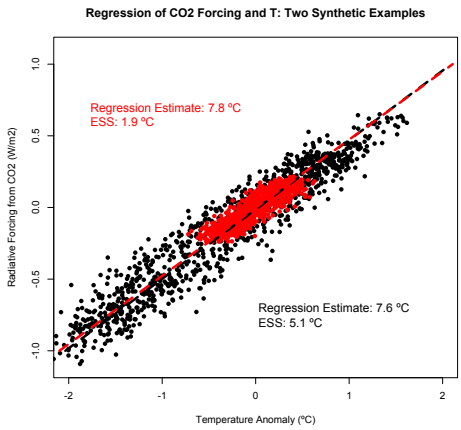

Figure S1: a) Scatterplot of temperature and CO_2 forcing for the two selected cases in Main text Fig. 1a.

the field would be near unity, instead, it varies widely and we conclude that ESS is almost always over-estimated by this procedure in the presence of independent forcing of the high latitude ice sheets.

Name	Description	Units	Value
a	Ice sheet sensitivity to temp.	mSL K^{-1}	0-30
b	Carbon cycle sensitivity to temp.	ppm K^{-1}	20
F_I	High latitude forcing of ice sheets	Wm^{-2}	$30 \sin(\pi t/\tau_F)$
H	Heat capacity of the system	J m^{-2} K^{-1}	N/A
μ	Ice sheet radiative forcing	Wm^{-2} mSL^{-1}	1/40
τ_C	Carbon cycle response time	kyr	3
τ_F	Glacial cycle half-period	kyr	40
τ_L	Ice sheet response time	kyr	10

Table S1: Description of model parameters and values.

References

1. Köhler, P. *et al.* What caused Earth's temperature variations during the last 800,000 years? Data-based evidence on radiative forcing and constraints on climate sensitivity. *Quat. Sci. Revs.* **29**, 129–145 (2010).