Psychometric functions for pure-tone frequency discrimination

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The form of the psychometric function (PF) for auditory frequency discrimination is of theoretical interest and practical importance. In this study, PFs for pure-tone frequency discrimination were measured for several standard frequencies (200–8000 Hz) and levels [35–85 dB sound pressure level (SPL)] in normal-hearing listeners. The proportion-correct data were fitted using a cumulative-Gaussian function of the sensitivity index, d', computed as a power transformation of the frequency difference, Δf. The exponent of the power function corresponded to the slope of the PF on $\log(d')$-$\log(\Delta f)$ coordinates. The influence of attentional lapses on PF-slope estimates was investigated. When attentional lapses were not taken into account, the estimated PF slopes on $\log(d')$-$\log(\Delta f)$ coordinates were found to be significantly lower than 1, suggesting a nonlinear relationship between d' and Δf. However, when lapse rate was included as a free parameter in the fits, PF slopes were found not to differ significantly from 1, consistent with a linear relationship between d' and Δf. This was the case across the wide ranges of frequencies and levels tested in this study. Therefore, spectral and temporal models of frequency discrimination must account for a linear relationship between d' and Δf across a wide range of frequencies and levels.
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I. INTRODUCTION

The function relating sensitivity (d') to the magnitude of the stimulus difference (Δ) in a psychophysical discrimination experiment is commonly referred to as the "psychometric function" (PF). Knowledge of the form of the PF has both theoretical implications and practical applications. For example, to meaningfully compare discrimination thresholds measured in different experiments, which may target different d', it is necessary to know how d' is related to Δ (Green and Swets, 1966; Jesteadt and Bilger, 1974; Jesteadt and Sims, 1975; Lyzenga, 1997). Ideally, the units in which Δ is expressed should be chosen in such a way that the relationship is linear, consistent with the equal-variance Gaussian model of signal detection theory (Green and Swets, 1966; Buus and Florentine, 1991; Macmillan and Creelman, 2005).

Because intensity and frequency discrimination are two of the most basic and most thoroughly studied auditory abilities, one might expect that the relationship between sensitivity and the relevant stimulus variable in these two tasks is well documented. Indeed, the form of the PF for intensity discrimination, and the related issue of the most appropriate metric for intensity-discrimination thresholds, have generated considerable interest (e.g., McGill and Goldberg, 1968; Durlach and Braida, 1969; Rabinowitz *et al.*, 1976; Zwislocki and Jordan, 1986; Laming, 1986; Hellman *et al.*, 1987; Viemeister, 1988; Viemeister and Bacon, 1988; Buus and Florentine, 1991; Dai and Green, 1992; Buus *et al.*, 1995; Shepherd and Hautus, 2007). Buus and Florentine (1991) showed that in a pure-tone intensity-discrimination experiment, d' is approximately proportional to the level difference, ΔL in dB, indicating that the latter is an appropriate metric for intensity-discrimination thresholds. Other authors have suggested that d' comes closest to being approximately proportional to $\Delta p/p$ (e.g., Laming, 1986; Shepherd and Hautus, 2007).

The form of the psychometric function for frequency discrimination has been less extensively explored. Although some investigators have concluded that d' is approximately proportional to the frequency difference, Δf (in Hz) between the tones being discriminated (Jesteadt and Bilger, 1974; Jesteadt and Sims, 1975; Wier *et al.*, 1976; Turner and Nelson, 1982; Nelson and Freyman, 1986), detailed examination of the evidence on which this conclusion is based reveals the following limitations. First, although the d'-versus-Δf data of Jesteadt and Bilger (1974) and Jesteadt and Sims (1975) appear to be remarkably "noiseless," and well fitted by straight lines (see Fig. 1 in each of these articles), it is important to note that the fits are based on measurements obtained at two Δf's only in the former study and three Δf's only in the latter study. The studies of Wier *et al.* (1976), Turner and Nelson (1982), and Nelson and Freyman (1986) involved more Δf's, but the data were visibly more variable than those of Jesteadt and Bilger (1974) and Jesteadt and Sims (1975)[1]. Second, in all of these studies, the d'-versus-Δf data were force-fitted with linear functions only, leaving open the possibility that the data could be better fitted by nonlinear functions. Careful examination of the example data in Turner and

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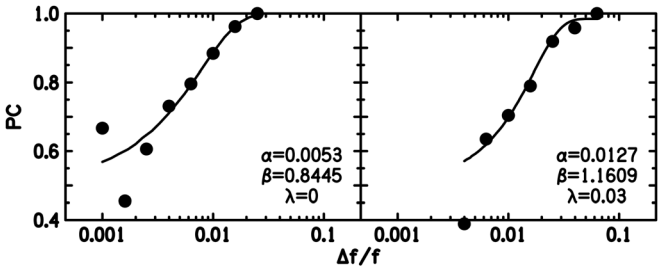

FIG. 1. Example PFs constructed using data from adaptive tracks. The filled circles show proportions of correct responses estimated for each value of the tracking variable (Δf) visited by the adaptive track. The solid lines through the data points show maximum-likelihood fits (see text for details). The estimates of the PF parameters, threshold (α), slope (β), and lapse rate (λ), are indicated within each plot. The offset of the asymptote in PC of the PF from $PC = 1$ is $\lambda/2$.

Nelson's (1982) article (see the filled circles in Fig. 1 in that article) and Nelson and Freyman's (1986) article (see the filled circles in Figs. 1 and 2 in that article) suggests that a concave function could have provided a better fit. However, it remains unclear whether the saturating trend apparent in these

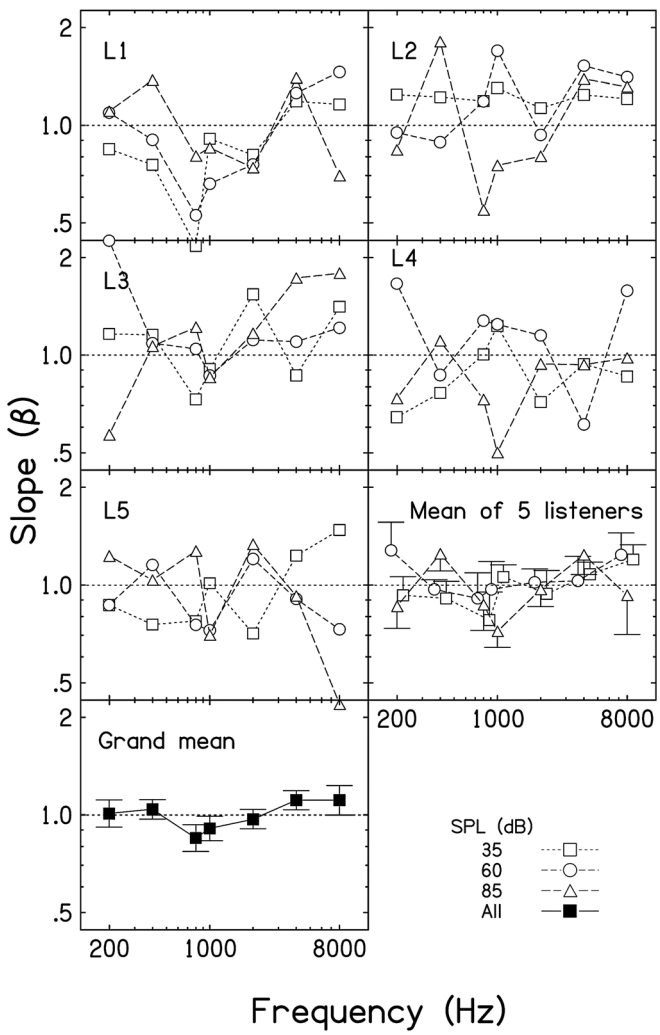

FIG. 2. Maximum-likelihood estimates of the slopes (β) of PFs. Panels L1–L5: data from individual listeners. Lower-right panel: mean across listeners for each sound level. Lower-left panel: grand mean obtained by averaging data across listeners and sound levels. The horizontal dashed lines indicate a slope of 1. Each error bar in the mean and grand-mean panels shows ± 1 standard error of the mean.

data reflects a ceiling effect due to lapses of attention or to the application of a correction for extreme values of proportions of correct responses in the d' calculations.

The only study (to our knowledge) in which the relationship between d' and Δf was fitted using a power-law function, thus accommodating linear as well as nonlinear relationships, indicated that the relationship was nonlinear (Dai, 1995). The main purpose of that study was to demonstrate that an adaptive tracking procedure was effective for measuring PFs. As an example of the application, the study included a small set of PFs from human subjects for frequency discrimination. In that study, proportion-correct (PC) data were obtained using a constant-stimulus or an adaptive-tracking procedure, and they were fitted using a cumulative-Gaussian probability function of d'. The latter function had no free parameter. The only two free parameters in the model were the exponent and "threshold" of the power-law function relating d' to Δf; this power function is described in the Methods section. The results showed that the exponents of the best-fitting power-law functions were consistently lower than 1. Because the exponent of the power-law function corresponds to the slope of the line relating d' to Δf on log-log coordinates, an exponent lower than 1 implies a nonlinear (concave) relationship between d' and Δf on linear coordinates. To avoid confusion, throughout the paper, linear coordinates are implied whenever we refer to the relationship between d' and Δf, unless noted otherwise. Because Dai (1995) found power-law exponents lower than 1 regardless of whether the fits were based on data obtained using a constant-stimulus procedure or on data obtained using an adaptive-tracking procedure, this outcome appears not to be specific to the use of an adaptive-tracking procedure.

Because the examples of measured PFs in Dai (1995) were from only two listeners, it is unclear whether the conclusions of this study generalize to other listeners. Moreover, fitting procedure used by Dai did not take into account the possible influence of attentional lapses on the estimates of the PF parameters. Using computer simulations, Wichmann and Hill (2001) demonstrated that it is important to take into account attentional lapses when estimating PF parameters. Their results indicate that when attentional lapses are present and unaccounted for in the fitting, the resulting estimates of PF slope are consistently lower than the slope of the true (underlying) PF.

The current study was undertaken to clarify the origin of the discrepancy between the conclusions of Dai's (1995) study and the conclusions of other previous studies, which found a linear relationship between d' and Δf. In particular, we sought to determine whether Dai's (1995) finding of power-law exponents consistently lower than 1 for the relationship between d' and Δf could be confirmed in a larger sample of listeners and using a PF-fitting procedure that takes into account attentional lapses. A second aim of the current study was to test for differences in the relationship between d' and Δf between low and high frequencies. It has been suggested that pure-tone frequency discrimination depends on temporal coding (phase-locking) below about 4 kHz but on "place" (tonotopic) coding at higher frequencies (Moore, 1973; for a review of arguments supporting this claim, see: Moore, 2003). Accordingly, one might expect a

different relationship between d' and Δf below and above 4 kHz. Of all the studies cited in the preceding text, only one measured PFs for frequency discrimination at frequencies higher than 4 kHz (Nelson and Freyman, 1986); unfortunately, the corresponding data were not reported.

II. METHOD

A. Listeners

Five listeners participated in the study. All had normal hearing, defined as pure-tone hearing thresholds of 20 dB hearing level (HL) or less at octave frequencies between 125 and 8000 Hz. The listeners, who were undergraduate students, had all participated in psychoacoustics experiments prior to the current study, and they had many hours of practice with the test equipment and with the adaptive-tracking procedure. During the experiments, the listeners were seated in a sound-attenuating room. All participants provided written informed consent prior to their inclusion in the study and were paid an hourly wage for their services.

B. Stimuli

The stimuli were pure tones shaped with 10-ms squared-cosine onset and offset ramps. The total duration of each tone, including rise and fall times, was 200 ms. This duration was chosen as a compromise between keeping the duration of each trial relatively short, and at the same time ensuring that thresholds would be comparable to those obtained in other studies of frequency discrimination, which have used longer durations. Data in the psychoacoustics literature indicate that little further improvement in frequency-discrimination thresholds with increasing tone duration occurs beyond about 200 ms (Micheyl *et al.*, 1998). The tones were presented in separate conditions at 35, 60, and 85 dB sound pressure level (SPL). The frequency of the standard tone, f, was equal to 200, 400, 800, 1000, 2000, 4000, or 8000 Hz. The frequency of the comparison tone was higher than the standard frequency by an amount Δf (in Hz), which was varied adaptively.

C. Procedure

PFs for pure-tone frequency discrimination were measured using a 2I-2AFC paradigm and a three-down one-up adaptive-tracking procedure (Levitt, 1971).[2] On each trial, the standard and comparison tones were presented in random order. The two observation intervals were marked by visual display "1" and then "2" on the computer monitor. The two tones on a trial were separated by 500 ms. Listeners were informed that one of the two tones presented on a trial would have a higher frequency ($f+\Delta f$) than the other tone (f). Their task was to indicate, by pressing one of two keys on a computer keyboard, in which observation interval the higher-frequency tone was presented. Each response was followed by visual feedback indicating whether the response was correct or not. Each listener performed 15 blocks of 60 trials (for a total of 900 trials) for each condition, where a "condition" means a given combination of standard frequency and sound level. At the beginning of the first block of each condition, Δf was set to a value approximately equal to 10 times the expected frequency difference limen so that the tones would be easily discriminable by the listener. For the 1000-Hz standard frequency, this value was equal to 2.51% of the standard frequency or 25.1 Hz. Following each incorrect response, Δf was increased by a factor of 1.585. After three consecutive correct responses, Δf was reduced by the same factor. For a given condition, and a given listener, the last Δf value on the just-completed block was used as the starting Δf for the next block; this was done so that adaptive tracks would converge more rapidly than if they had started at a fixed Δf value, which was unnecessarily large for the considered listener and condition. Upon completion of a block, the listener initiated the next block by pressing the "enter" key.

D. Apparatus

The signals were generated digitally and played through a 16-bit digital-to-analog converter (TDT DA 1) at a sampling rate of 25 kHz, and low-pass filtered at 10 kHz (TDT FLT3). The sounds were presented diotically to the listeners via Sennheiser (HD450) headphones.

E. Data analysis

All trials performed by a given listener in a given condition were sorted based on the level of the tracking variable, Δf. For each Δf, the number of trials on which the response was correct was counted, and this number was divided by the total number of trials at that Δf to obtain an estimate of PC for that Δf. The resulting PC values were fitted using a scaled cumulative Gaussian:

$$PC = \frac{1}{2}\lambda + (1-\lambda)\Phi(d'/\sqrt{2}) \tag{1}$$

where λ denotes the lapse-rate, and Φ denotes the cumulative-normal function. The first term reflects that the performance is at chance (i.e., 50%) on the proportion of trials (λ) when the listener is not attending to the tasks. The second term reflects that the performance is described by $\Phi(d'/\sqrt{2})$ on the proportion of trials $(1-\lambda)$ when the listener is fully attending to the tasks. [An expression similar to Eq. (1) has been used by Wichmann and Hill, 2001]. As in Dai (1995), the sensitivity index d' is related to the stimulus variables by

$$d' = \left(\frac{\Delta f/f}{\alpha}\right)^{\beta}, \tag{2}$$

where f denotes the frequency of the standard tone (in Hz) and Δf denotes the frequency difference between the standard and comparison tones (also in Hz). The exponent, β, corresponds to the slope of the PF on log-log coordinates. If $\beta=1$, Eq. (2) reduces to a linear relationship between Δf and d', and it traces a straight line with a slope of 1 on $\log(d')$-$\log(\Delta f)$ coordinates. The parameter α represents the "threshold," which is defined as the value of $\Delta f/f$ corresponding to $d'=1$. Geometrically, α is the x intercept of the straight line traced by Eq. (2) on $\log(\Delta f/f)$-$\log(d')$ coordinates. Numerous studies have used forms similar to Eq. (2)

for describing PFs (e.g., Green and Swets, 1966; Egan *et al.*, 1969; Buus *et al.*, 1986; Laming, 1986, p. 45; Dai, 1994, 1995; Klein, 2001; Wichmann and Hill, 2001).

The three fitting parameters, α, β, and λ, could only take positive values. The lapse-rate parameter, λ, was not allowed to exceed 0.40, reflecting the assumption that listeners were never inattentive on more than 40% of the trials. Equation (1) was fitted to the PC data from the adaptive procedure using a maximum-likelihood algorithm. The likelihood of the data (which consisted of the ensemble of PC values for a given listener and condition) given the model was computed as

$$L[p_1, p_2, \ldots, p_m] = \prod_{i=1}^{m} P[p_i(n_i)|(\alpha, \beta, \lambda)], \tag{3}$$

where $P[p_i(n_i)|(\alpha, \beta, \lambda)]$ denotes the probability of observing a PC value based on n_i trials corresponding to the ith value of Δf, conditioned on the model parameters. Equation (3) follows directly from the assumption of conditional independence between the p_i's, which implies (by the definition of independence) that the joint probability density function of the p_i's (conditioned on the parameter values) can be written as the product of the conditional marginal probability density functions of the p_i's (see, e.g., Green, 1990; Wichmann and Hill, 2001). The three parameters α, β, λ were estimated, using a simplex method, as the values that maximized the likelihood, $L[p_1, p_2, \ldots, p_m]$.

Figure 1 shows two examples of PFs constructed by counting the number of correct responses at each Δf visited by the three-down one-up adaptive-tracking procedure, and the best-fitting (maximum-likelihood) functions defined by Eqs. (1) and (2). The PC values shown in the left- and right-hand panels of Fig. 1, and the number of trials on which these values are based, are listed in Table I. These examples are from different listeners and different stimulus conditions. The data shown in the left-hand panel were obtained using 35 dB SPL tones and a standard frequency of 200 Hz. The data shown in the right-hand panel were obtained using 35 dB SPL tones and a standard frequency of 8000 Hz. These two examples were chosen because they are representative of the data and fits that were obtained in other conditions and listeners. The left-hand panel illustrates a case in which the lapse rate was equal to 0. The right-hand panel illustrates a case in which the lapse rate was 0.03. The maximum-likelihood estimates of the PF parameters, threshold, slope, and lapse rate are indicated within each plot. The deviations (or "errors") between the measured PCs and the corresponding PCs "predicted" by the best-fitting PF, which are illustrated in these two plots, are typical; they fall within the range of binomial variability.

TABLE I. Example PF data collected using the three-down, one-up adaptive-tracking procedure. The data listed in this table correspond to those plotted in the left-hand panel of Fig. 2. The first column lists values of the relative change in frequency, expressed in $\Delta f/f$ in %, visited by the adaptive-tracking procedure, from the largest (at the top) to the smallest (at the bottom). In this example, the frequency of the standard tone was fixed at $f = 200$ Hz. The second column shows the PC values obtained computed by counting the number of correct responses across all trials corresponding to a given Δf. The last column lists the number of trials at each Δf.

$\Delta f/f$ (%)	PC	N trials
2.51	1.00	15
1.58	0.96	130
1.00	0.88	233
0.63	0.80	260
0.40	0.73	182
0.25	0.61	66
0.16	0.46	11
0.10	0.67	3

III. RESULTS

Figure 2 shows individual and mean PF slopes (β) as a function of the standard frequency, with stimulus level as a parameter. These data were analyzed using a two-way (frequency × level) repeated-measures analysis of variance (ANOVA) on the logarithm of the slopes. The reason for using the logarithm of the slope as the dependent variable in this analysis is that slope values are bounded below by 0, so that their distribution is skewed. In contrast, the distribution of the logarithm of the slopes is more symmetric and approximately Gaussian. The results showed no significant main effect of stimulus level [$F(2, 8) = 0.984$, $p = 0.414$], no significant main effect of frequency [$F(6, 24) = 2.029$, $p = 0.182$], and no significant interaction [$F(12,48) = 1.293$, $p = 0.253$]. Moreover, the slopes did not differ across the five listeners [$F(1,4) = 0.148$, $p = 0.720$]. On average, the slopes were not significantly different from 1 [$t(104) = -0.132$, $p = 0.108$]. These results are consistent with the hypothesis that d' is related linearly to Δf.

Figure 3 shows the individual and mean PF thresholds (α). These are in line with the frequency-discrimination thresholds measured using comparable frequencies and stimulus levels in trained listeners in previous studies (e.g., Moore, 1973; Wier *et al.*, 1977; Nelson *et al.*, 1983; Demany, 1985; Moore and Glasberg, 1989; Dai *et al.*, 1995; Delhommeau *et al.*, 2005; Amitay *et al.*, 2006; Micheyl *et al.*, 2006). The smallest mean threshold, expressed as $\Delta f/f$ in percent, was observed for the 1000-Hz standard frequency; it was equal to 0.2%. The good agreement between the thresholds measured in the current study and the thresholds reported in previous studies provides further indication that our use of an adaptive-tracking procedure for measuring PFs, and the fitting technique that we used to estimate the PF parameters, are adequate.

Figure 4 shows the distribution of $\lambda/2$ (the amount by which the asymptotic PC of the best-fitting PF was lower than 1) for each listener, and the overall distribution for all five listeners. These plots make it apparent that the listeners differed in their ability to maintain attention throughout the tests. Listener L1 appeared to be the most attentive. For this listener, $\lambda/2$ was always below 0.06, and for half of the conditions, it was 0. For the other listeners, $\lambda/2$ sometimes exceeded 0.1. Overall, the median lapse rate was equal to 0.034. This is similar to the median lapse rate for the adult listeners in the study of Buss *et al.* (2009), which measured PFs for pure-tone intensity discrimination.

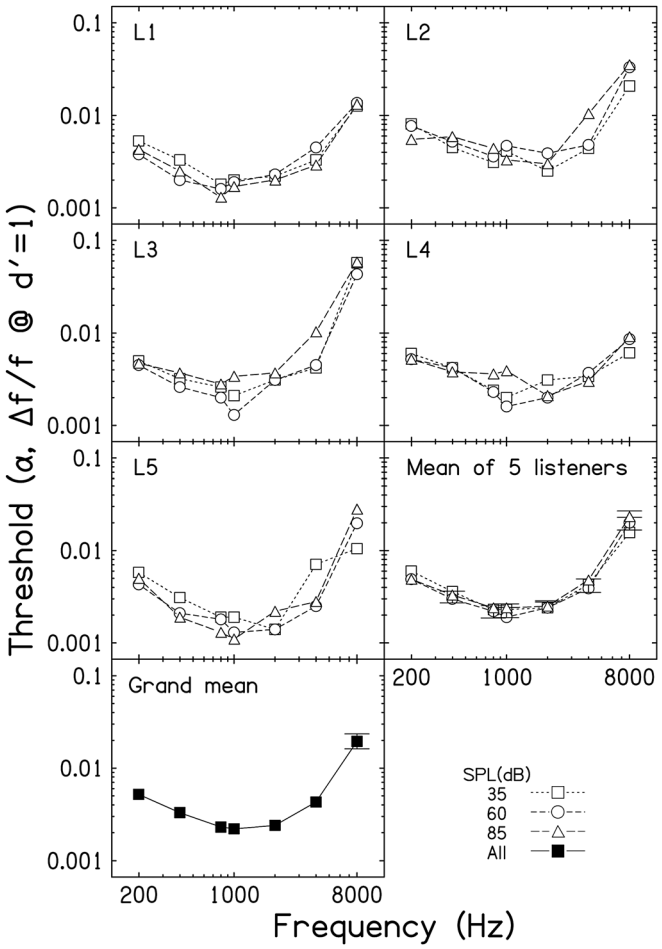

FIG. 3. Thresholds (α): Panels L1–L5: data from individual listeners. Lower-right panel: mean across listeners for each sound level. Lower-left panel: grand mean obtained by averaging data across listeners and sound levels. The horizontal dashed lines indicate a slope of 1. Each error bar in the mean and grand-mean panels shows ± 1 standard error of the mean.

For nearly one-third of the total 105 PFs examined in the present study, the estimated lapse rate was zero. For these cases, the slope and threshold estimates remained unchanged when the fits were recomputed using only two free parameters (α and β) as in Dai's (1995) study. For the remaining two-thirds of cases (71), including the lapse rate as a free parameter when fitting PFs led to smaller mean squared errors between the measured and predicted (best-fitting) PFs. For more than half of these cases (41 of 71), the difference in mean squared error between the two- and three-parameter models was statistically significant (nested χ^2 test: $\chi^2(1) \geq 3.84$, $p < 0.05$).

The results in Fig. 4 should be of interest to researchers in this field because they reflect the lapse rates that may be expected in adaptive-tracking measurements of frequency-discrimination thresholds in trained, adult listeners. Although previous studies have investigated effects of inattention on PFs for pure-tone detection and gap detection in children or adults (e.g., Wightman *et al.*, 1989; Bargones *et al.*, 1995; Werner *et al.*, 2009), or using computer simulations (e.g., Viemeister and Schlauch, 1992; Green, 1995), to our knowledge, the existing psychoacoustic literature contains no indication of the distribution of lapse rates in adult listeners during adaptive-tracking measurements of frequency discrimination.

IV. DISCUSSION

The results of this study are broadly consistent with the hypothesis that the relationship between d' and Δf (for Hz) in pure-tone frequency discrimination is linear. These results provide further support to the conclusions of earlier studies (Jesteadt and Bilger, 1974; Jesteadt and Sims, 1975; Wier *et al.*, 1976; Turner and Nelson, 1982; Nelson and Freyman, 1986), which also measured linear PFs for pure-tone frequency discrimination. One important difference between the current study and these earlier studies is that in the earlier studies, the d'-versus-Δf data were fitted using only linear functions. The exclusive use of linear functions when fitting psychometric data is equivalent to imposing a strong implicit prior that favors linear PFs over nonlinear ones, and it leaves open the question of whether other functions would fit the data equally well—and perhaps, better. In contrast, in the current study, the relationship between d' and Δf was modeled using a flexible power function. The exponent of the power function was a free parameter, which could take any value—including, but not limited to, 1. Therefore the finding that, on average, the exponent was generally close to—and not significantly different from—1, provides stronger evidence for the "linear relationship" hypothesis than the results of previous studies in which only linear fits were tested.

A second noteworthy difference between the current study and these previous studies relates to the use of a principled approach to the fitting of psychometric data in the current study. In the previous studies, the measured PCs were first transformed into d' (*separately from the other PCs measured using other* Δf*'s in the same listener*), and the resulting d'-versus-Δf data were fitted using a least-squares criterion. In contrast, in the current study, PC estimates derived from adaptive tracks were fitted directly and as a whole (including all Δf's visited by the tracking procedure) using a non-linear (cumulative-Gaussian) function and a maximum-likelihood criterion. The latter approach avoids the need to transform measured PCs "point-wise" into d'—which can occasionally result in undefined values when PC is at ceiling, requiring the use of a correction for extreme proportion, which can bias the results (Hautus, 1995) and does not completely eliminate the ceiling effect. PCs at ceiling pose no particular difficulty for the maximum-likelihood fitting procedure because, under the binomial distribution, counts equal to the number of trials have a non-zero probability of occurring even if the underlying probability of a correct response (determined by the PF) is less than 1 at that Δf. Another advantage of the maximum-likelihood fitting procedure is that differences in the relative reliability of PC estimates due to different numbers of trials across levels of the dependent variable (Δf) were automatically taken into account. The reason for this is that the variance of the binomial distribution, on which the maximum-likelihood was based, is inversely related to the number of trials. Because probability distributions are normalized in such a way that their area remains constant, independent of their variance, a wider spread of the probability mass implies a lower

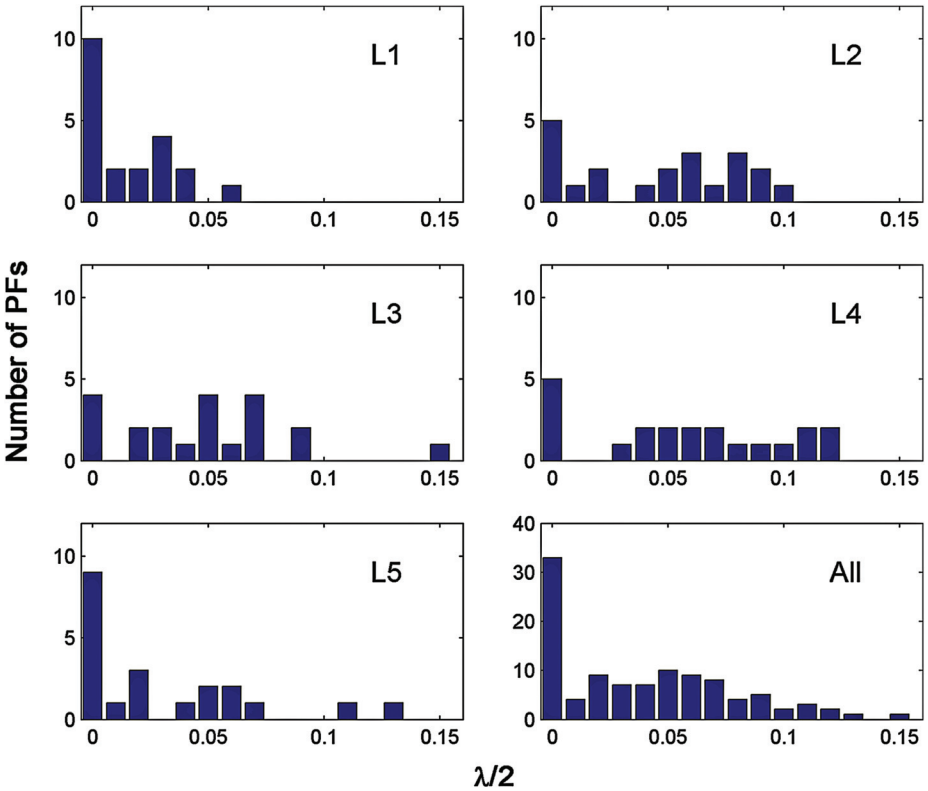

FIG. 4. (Color online) Individual and overall distributions of the estimates of the parameter for the lapse rate (λ). Note that, for a given lapse rate of λ, the psychometric function has an upper asymptote of $1-\lambda/2$. Because $\lambda/2$ is the offset of the asymptote PC of the psychometric function from a $PC = 1$, it is directly visible on the fitted PF. For this reason, we use $\lambda/2$ rather than λ in the abscissa.

likelihood for each observation than for a distribution whose probability mass is more concentrated (see: McKay, 2003, p. 344.)

A third important difference between the current study and previous studies of PFs for frequency discrimination is that, here, the PF-fitting procedure included an attentional lapse-rate parameter. Attentional lapses can produce an apparent saturation in PFs even when, in reality, the underlying PF is linear (Wichmann and Hill, 2001). As mentioned in the Introduction, Dai (1995) reported slope (β) values consistently lower than 1 for PFs for the frequency discrimination of 1-kHz pure tones in two normal-hearing listeners. Unlike the procedure used in the current study, the fitting procedure used by Dai (1995) did not include a free parameter for the lapse rate. To determine whether this difference between Dai's (1995) study and the current one could account for the discrepant conclusions of these two studies concerning the linearity of PFs for frequency discrimination, the data of the current study were reanalyzed using a constant lapse rate of zero. Using this procedure, the mean estimated slope was found to be 0.77, significantly lower than 1 [$t(104) = 9.52, p < 0.00001$]. For the same standard frequency as used by Dai (1995), the mean slope estimate determined using a zero lapse rate in the current study was equal to 0.73. This is within the range of slopes (0.69-0.85) reported by Dai (1995).

The preceding result demonstrates a potential bias in the estimation of PF slopes. Estimates of PF slopes can be significantly lower when attentional lapses are not taken into account than when they are. The reason for this effect is as follows. If the function relating d' to PC has an upper asymptote of 1, but due to attentional lapses the measured PCs never reach 1 (even for large Δf's), a shallower slope is needed to minimize the error between the predicted PCs and the measured PCs at large Δf's than if the upper asymptote of the function is allowed to take values lower than 1. These considerations underline the importance of taking attentional lapses into account when fitting PFs. This caveat applies regardless of whether PCs are measured using an adaptive-tracking procedure or a constant-stimulus procedure. Moreover, it is not specific to the use of a maximum-likelihood fitting algorithm because we observed a similar effect when we re-computed the fits using an algorithm that minimized the chi-square error between measured and predicted PCs. It will be interesting, in future studies, to verify that lapse-rate estimates obtained from PF fits concord with other measures of listeners' inattention, obtained using another approach (e.g., confidence judgments or, possibly, noninvasive neural measures such as long-latency auditory event-related-potentials).

In addition to providing further evidence for a linear relationship between d' and Δf in pure-tone frequency discrimination, the results of this study indicate that this linear relationship holds over a wide range of frequencies (from 200 to 8000 Hz) and stimulus levels (from 35 to 85 dB SPL). In the following sections, we discuss some of the implications of this finding for computational models of frequency discrimination of pure tones and fundamental-frequency discrimination of complex tones.

A. Implications for models of pure-tone frequency discrimination

A secondary aim of this study was to test for possible differences in the form or slope of PFs for frequency

discrimination between frequencies lower than 4 kHz and higher frequencies. The results showed no significant difference in PF slopes across frequency over the 200-8000 Hz range. However, this does not necessarily imply that pure-tone frequency discrimination involves the same mechanisms at low and high frequencies. As explained in the following text, both time-based (phase-locking) models and place-based (tonotopic) models of frequency discrimination can, in principle, account for a linear relationship between d' Δf.

"Place" models of frequency discrimination posit that pure-tone frequency discrimination depends on the detection of differences in excitation level at the output of auditory filters with center frequencies located on the upper or lower side of the peaks in the excitation patterns produced by the tones (Henning, 1967; Zwicker, 1970; Dai *et al.*, 1995; see also Moore, 2003, for an introduction to the use of excitation patterns in auditory research). According to such models, therefore, frequency discrimination involves the same type of cue as intensity discrimination: a change in excitation level. PFs for pure-tone intensity discrimination have been found to be approximately linear when d' is plotted as a function of the level difference (ΔL in dB) between the standard and comparison tones (e.g., Buus and Florentine, 1991). If frequency and intensity discriminations for pure tones ultimately rely on the detection of a difference in excitation level, and the difference in excitation level is proportional to Δf (Dai *et al.*, 1995), a linear relationship between d' and Δf should be expected.[3] Henning's (1967) and Zwicker's (1970) single-filter models of frequency discrimination suffer from important shortcomings. In particular, these models cannot account for the finding that varying the level of each tone randomly over a wide range has a relatively small impact on frequency discrimination thresholds (Dai *et al.*, 1995). The level randomization would render the change at the output of any single filter an unreliable cue for detecting the change in the frequency of the tone. However, this problem for the single-filter models is easily remedied by assuming that the auditory system can combine changes in excitation at the output of two auditory filters, one with a center frequency below the signal and another with a center frequency above the signal (Dai *et al.*, 1995). As long as differences in excitation level are combined linearly, the PF for frequency discrimination predicted based on the assumption of a linear dependence of d' on differences in excitation level should also be linear.

The second main type of model that has been proposed for auditory frequency discrimination involves comparisons of interspike intervals (e.g., Siebert, 1968, 1970; Luce and Green, 1974; Hanekom and Kruger, 2001; Heinz *et al.*, 2001; see also Rose *et al.*, 1967, and McKinney and Delgutte, 1999, for the statistical properties of inter-spike intervals). The model proposed by Luce and Green (1974), in particular, predicts an approximately linear relationship between d' and Δf.[4] To summarize, excitation-pattern and temporal models of frequency discrimination are both consistent with a linear relationship between d' and Δf.

B. Implications for models of complex-tone fundamental-frequency discrimination

The current findings also have implications for theories and models of the perception of the pitch of harmonic complex tones. Several models of pitch perception assume that "virtual" or "periodicity" pitch, which usually corresponds to the fundamental frequency (f_0) of a complex tone, is determined based on the frequencies of individual harmonics, or noisy estimates of these frequencies (Terhardt, 1970; Goldstein, 1973; Wightman, 1973). In Goldstein's (1973) optimal-processor model, the accuracy and precision with which f_0—and thus, virtual pitch—is estimated is directly related to the accuracy and precision with which the frequencies of individual components are estimated. The optimal estimate of f_0 is related to the estimates of the individual frequencies by

$$\frac{\hat{f}_0}{\sigma_0^2} = \sum_{k=1}^{n} \frac{\hat{f}_k}{k\sigma_k^2}. \tag{5}$$

In this equation (Dai, 2000), $\hat{f}_0$ and $\hat{f}_k$ denote the estimates, or the internal representations, of the f_0, and of the frequency of the kth component, respectively; σ_0^2 and σ_k^2 denote the variance of the estimates of f_0 and f_k; and n is the number of components. Equation (5) implies a linear relationship between the sensitivity (d') and the change in f_0 (Δf_0). This can be seen by noting that estimates of f_0 obtained using this equation are linear combinations of the estimates of the frequencies of individual components. By definition, d' for discriminating the frequency of the kth component is proportional to $\Delta\hat{f}_k$. Since this d' is proportional to Δf_k (as shown by the results of the present study), it follows that $\Delta\hat{f}_k$ is proportional to Δf_k and thus to Δf_0 (because $f_k = kf_0$). Likewise, by definition, d' for the discriminating the fundamental frequency is proportional to $\Delta\hat{f}_0$. Because $\Delta\hat{f}_0$ is proportional to $\Delta\hat{f}_k$ [Eq. (5)] and thus also to Δf_0 (see above), it follows that the d' for f_0 discrimination is proportional to Δf_0. Thus, the PF for fundamental-frequency discrimination is also linear.

That the PF for fundamental-frequency discrimination is linear can also be shown as follows. According to a multiband model of signal-detection theory, the PFs of individual bands are parallel to the PF based on a decision variable that is a linear combination of the outputs of the individual bands (Green and Swets, 1966; Buus *et al.*, 1986). Mathematically, Eq. (5) has the same form as that given by the multiband model in which the representation of the fundamental frequency is a linear combination of the representations of the frequencies of individual components. Thus, it follows that the PF for f_0 discrimination is linear relative to Δf_0. In short, Goldstein's (1973) model predicts that if d' for the discrimination of component frequencies is proportional to Δf, d' for the discrimination of f_0 should be proportional to Δf_0.

Findings consistent with the prediction of a linear relationship between Δf_0 and d' in f_0 discrimination were reported by Plack and Carlyon (1995). These authors measured PFs for f_0 discrimination in four listeners. They found that d' was approximately proportional to Δf_0, as indicated by near-unity slopes of best-fitting straight lines through the

data on log(d')-log(Δf_0) coordinates. Interestingly, Plack and Carlyon tested stimulus conditions that involved "resolved" harmonics—for which the perception of f_0 requires a combination of information on the frequencies of individual harmonics—and conditions that involved only "unresolved" harmonics—for which the perception of f_0 may not have a simple relationship to changes in the frequency of individual harmonics. The finding of linear PFs even in the latter condition indicates that whichever cue is used to discriminate pitch in the absence of resolved harmonics, such as differences in modulation rate (Burns and Viemeister, 1976) or in the time intervals between peaks in the waveform fine structure (de Boer, 1956; Schouten *et al.*, 1962; Moore *et al.*, 2009), sensitivity to this cue is approximately proportional to the change in fundamental frequency.

C. Does the linear relationship extend to large values of $\Delta f/f$?

It is important to note that the proportionality relationship between $\Delta f/f$ and d', which was observed in the current study and in previous ones, was established for relatively small values of $\Delta f/f$, corresponding to Δf of, e.g., a few hertz for a standard frequency of 1000 Hz. We do not expect that this relationship extends to large Δf's. A linear relationship between d' and $\Delta f/f$ would appear to imply that the pitches of two pairs of tones separated by the same frequency difference (in hertz or as a proportion of the lower-tone frequency) should be perceived as equally distant. However, most musical scales are based on that notion that frequency changes that are equal in octaves are perceptually equivalent (Burns and Ward, 1998), and the results of several psychoacoustical studies indicate that the perception of pitch intervals depends more closely on frequency *ratios* than on arithmetic frequency *differences* (Attneave and Olson, 1971; Greenwood, 1997; Russo and Thompson, 2005). For instance, pairs of tones separated by a constant frequency ratio (e.g., one octave) are perceived as roughly equidistant in pitch—although listeners appear to "stretch" octaves at higher frequencies (Stevens *et al.*, 1937; Stevens and Volkmann, 1940; Ward, 1954).

The proportionality relationship between d' and $\Delta f/f$ observed in PF measurements may appear to be inconsistent with the logarithmic relationship between pitch and frequency observed in judgments of musical interval size. However, the inconsistency can be resolved simply by noting that the latter experiments involved large values of $\Delta f/f$ (larger than a semitone, or 6%), whereas PF measurements typically involve very small values of $\Delta f/f$ (usually less than 6%, and sometimes smaller than 1%). Over a very small range of $\Delta f/f$, the logarithmic function is well approximated by a linear function.[5] Consequently, for small values of $\Delta f/f$, it is very difficult to distinguish linear and logarithmic PFs especially given the variability inherent in psychophysical measures. Therefore, the finding of a linear relationship between d' and $\Delta f/f$ should not be interpreted as evidence that differences in pitch between pure-tones are precisely proportional to $\Delta f/f$ and that frequency differences are a more "correct" metric than octave for measuring the perception of pitch distance. It is simply that, for practical purposes, the difference between the two scales is negligible at small values of $\Delta f/f$.

V. SUMMARY

Psychometric functions for pure-tone frequency discrimination were measured in five normal-hearing listeners using an adaptive-tracking procedure over a wide range of standard frequencies (200–8000 Hz) and stimulus levels (35–85 dB SPL). The data were fitted using a power-law function, which could accommodate both linear and nonlinear relationships between d' and Δf. Moreover, the rate of attentional lapses was included as a free parameter in the fits. The results indicate that d' for pure-tone frequency discrimination is related linearly to Δf (in Hz). In addition, the results indicate that attentional lapses can have a significant influence on estimated PF parameters. Excitation-pattern and temporal (phase-locking) models of frequency discrimination can both account for the linear relationship between d' and Δf. Therefore, the finding that this linear relationship exists at both low (<4 kHz) and high (>4 kHz) frequencies does not contradict the hypothesis that frequency-discrimination involves different mechanisms at low and high frequencies.

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[1]An article by Henning (1966), which was not included in the list of articles showing a linear relationship between d' and Δf, but which deserves mention, reported complete PF data for one listener. We digitized these data, transformed them to d', and fitted the d' values from each condition with a straight line on log(Δf)-log(d') coordinates. The resulting slopes were found to vary from 0.75 (for the 4-kHz fixed-level condition) to 1.65 (for the 1-kHz fixed-level condition), with a geometric mean of 1.13. Unfortunately, Henning's article does not contain enough information concerning the variability of the measurements for us to determine whether these slope values differ statistically significantly from 1.

[2]It has been pointed out that estimates of PF slopes based on data from adaptive tracks can show a bias if the number of trials is small (Leek *et al.*, 1992; Dai, 1995; Treutwein and Strasburger, 1999; Kaernbach, 2001; see also Ulrich and Vorberg, 2009, for other pitfalls of 2AFC tasks). However, using a combination of computer simulations and comparisons between empirical estimates of slopes obtained using both constant-stimulus and adaptive-tracking procedures in the same listeners, Dai (1995) showed that the bias of PF-slope estimates obtained under conditions similar to those used in the current study, based on a large number of trials, is negligible.

[3]For PFs for frequency discrimination, a slope of unity can be predicted by the excitation-pattern (EP) model, as follows. According to the EP model, a change in frequency, Δf, is detected based on a corresponding change in excitation, ΔL. These two quantities are related via the slope (s_{EP}) of the steepest section of the EP, by $\Delta f = \Delta L / s_{EP}$. Buus and Florentine (1991) have shown, based on extensive data, that the PF for level discrimination is well described by $d' = (\Delta L/\alpha_L)^{\beta_L}$, with $\beta_L \approx 1$. Assuming that the PF for frequency discrimination is given by $d' = (\Delta f/\alpha_f)^{\beta_f}$, it follows from the EP model that $\beta_f = \beta_L \approx 1$.

[4]Based on the timing model by Luce and Green (1974, third equation of Appendix C), the sensitivity index d', is proportional to $(1 - f/f + \Delta f)$, which can be expressed as $(\Delta f/f + \Delta f)$. Because $\Delta f << f$, the preceding model predicts that d' is approximately proportional to $(\Delta f/f)$.

[5]The argument is as follows. Suppose that equal frequency ratios are perceived as equal pitch intervals, consistent with the use of "logarithmic" (e.g., octave) frequency scales in music. On the octave scale, the PF for frequency discrimination can be expressed as $d' = kx_{octave}$, where $x_{octave} = \log_2(1 + \Delta f/f) = \ln(1 + \Delta f/f)/\ln(2)$. The term $\ln(1 + \Delta f/f)$ can be expanded into a Taylor series: $\ln(1 + \Delta f/f) = \Delta f/f - \frac{1}{2}(\Delta f/f)^2 + \frac{1}{3}(\Delta f/f)^3 \cdots$. Typically, in frequency-discrimination experiments, $\Delta f/f << 1$. Therefore, we can ignore higher order terms in the Taylor series to obtain: $\ln(1 + \Delta f/f) \approx \Delta f/f$. Accordingly, for $\Delta f/f << 1$, the PF for frequency discrimination is approximately: $d' \approx k/\ln(2)\Delta f/f$. This shows that a linear relationship between d' and x_{octave} is not inconsistent with empirical findings of an approximately linear relationship between d' and $\Delta f/f$.

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