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CONCURRENT RANDOM-INTERVAL SCHEDULES AND THE MATCHING LAW[1]

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In Experiment I, a group of eight pigeons performed on concurrent random-interval schedules constructed by holding probability equal and varying cycle time to produce ratios of reinforcer densities of 1:1, 3:1, and 5:1 for key pecking. Schedules for a second group of seven were constructed with equal cycle times and unequal probabilities. Both groups deviated from simple matching, but the two forms of the schedules appeared to produce no consistent patterns of deviation. The data were found to be consistent with those obtained in concurrent variable-interval situations. The parameters of the matching equation in the form of $Y = k X^a$ were estimated; the value of k was unity and a was 0.84. In Experiment II, six pigeons were exposed to two *conc* RI RI schedules in which one component increasingly approximated an FI schedule. The value of k was not 1.0. Concurrent RI RI schedules were shown to represent a continuum from *conc* FI VI to *conc* VI VI schedules. The use of the exponential equation in testing "matching laws" suggests that $a < 1$ will continue to be observed, and this will set limits on the form of new laws and the assumed or rational values of the component variables in these laws.

Key words: concurrent schedules, matching law, random-interval schedules, key pecks, pigeons

Herrnstein's (1961, 1970, 1974) matching law is an elegant quantitative formulation of a relation between response distributions and reinforcer distributions in concurrent variable-interval variable-interval (*conc* VI VI) schedules of reinforcement. It is written as:

$$B_1/(B_1 + B_2) = r_1/(r_1 + r_2). \quad (1)$$

Here, B_1 and B_2 are counts of responses on alternatives 1 and 2; r_1 and r_2 are counts of reinforcers for responses on alternatives 1 and 2. An algebraic equivalent of Equation 1 is:

$$B_1/B_2 = r_1/r_2. \quad (2)$$

Several experimenters (Baum, 1974; de Villiers, 1977; Staddon, 1977) have found that Equation 2 is often not satisfactory. They have considered a more general formulation:

$$B_1/B_2 = k(r_1/r_2)^a. \quad (3)$$

As the latter writers have shown, a large amount of data from experiments employing *conc* VI VI schedules are better accounted for by Equation 3 than by Equation 2. This result is not surprising. An equation with two free parameters will often fit data better than an equation with no free parameters. Equation 2 is a powerful simplification of a relation between two variables of obvious importance in operant conditioning: it is an empirical theory. As a theory, it implies that the values of k and a in Equation 3 are unity. The values of k and a obtained by curve fitting for Equation 3 can be used to test the theory represented by Equation 2. To date, the values of k and a for individual organisms sometimes deviate from unity (Baum, 1974). Thus, the theory is sometimes brought into question.

If curve fitting by the method of least squares is used to estimate the parameters of Equation 3 from data for individual organisms, and these parameters are used to test the theory represented by Equation 2, then it is unlikely that k and a will be precisely unity in any particular case. At some arbitrary level of precision, Equation 2 is probably false in individual cases. How large can deviations from 1.0 be before falsification is accepted? How many "falsifying" samples among "confirming" samples must exist before a general evaluation can be seriously entertained? Given the elegance of Equation 2, its nomothetic po-

[1]The author wishes to thank Paul Donn for his help in conducting the experiment. Reprints may be obtained from the author, Department of Psychology, Central Michigan University, Mt. Pleasant, Michigan 48859.

tential should be examined thoroughly before replacing it with the less falsifiable Equation 3. If Equation 3 produces good fit to data from individual animals, then the resultant parameter values can be subjected to statistical methods for parameter estimation in the presence of random fluctuation, and the results can be used as a strong aid to intuition in the final evaluation of the nomothetic character of Equation 2.

The first experiment reported here was designed to test Equation 2 through parameter estimation for Equation 3. The schedules were concurrent random-interval random-interval (*conc* RI RI), rather than *conc* VI VI schedules. As will be shown later, available data for the latter schedules seriously question the validity of Equation 2. No data on *conc* RI RI schedules exist for this purpose.

Before proceeding with the experiment, the formal properties of VI and RI schedules may be profitably compared. With VI schedules, one generates a set of interreinforcer intervals with the constraint that the mean of the set has some desired value. The constituent intervals are then mixed and programmed, often via a punched tape device, for control of reinforcer delivery. The RI schedule has greater constraint in the intervals that occur. To obtain an RI schedule, one fixes an interval, T. A probability generator provides reinforcer availability at the end of T with probability P. If P is 1.0, then a fixed-interval (FI) schedule results no matter what T is. If P is less than 1.0, then the schedule is an RI no matter what T is. If P is less than 1.0 and the generator does not make a reinforcer available at the end of T, then another period of length T passes before a reinforcer is available, again with probability P. Since T and P can be chosen independently over a large range of values, a variety of mean interreinforcer times can be arranged. The mean interval is given by T/P and the variance of these intervals is $(T/P)^2(1-P)$ (Millenson, 1963). For VI schedules, the distribution of intervals can be of any kind, often rectangular, while for RI schedules the distribution approximates the geometric distribution. If behavior were insensitive to the shape of the distributions, then *conc* RI RI and *conc* VI VI results should be the same. However, if the interreinforcer distribution were an effective differentia, then Equation 2 might be supported. In either case, the generality of disconfirmation would be placed in perspective.

EXPERIMENT I

Method

Subjects

Fifteen experimentally naive Silver King pigeons were maintained at 80% to 85% of their free-feeding weights throughout the experiment.

Apparatus

Four Lehigh Valley Electronics pigeon chambers were individually housed in Industrial Acoustics Company isolation chambers. The operant conditioning boxes were supplied with a horizontal row of three transparent response keys, which could be illuminated from the rear with white light. The keys were 2.54 cm in diameter and were separated, center-to-center, by 8.25 cm. A grain dispenser was located below the second key and was illuminated when activated. Control of lights, programming of schedules, and data retrieval were effected by a Lehigh Valley Electronics Interact System employing a PDP 8/I 8k computer. No houselight was used. The apparatus is described in detail elsewhere (Rodewald, 1972).

Procedure

The pecking response to the center white key was shaped by the method of successive approximations. Two sessions followed with a random-ratio eight schedule in effect on the center key. The schedule was then changed to an RI 32-sec for two 30-min sessions. Next, the left and right keys, but not the center one, were alternately illuminated with white light during a 30-min session. The RI 32-sec schedule was in force for pecks on the lighted key with extinction (EXT) programmed on the remaining, dark keys. Exposure to the alternating RI 32-sec EXT was followed by two 30-min sessions on *conc* RI 32-sec RI 32-sec, with left and right keys simultaneously illuminated. For this schedule and for all other concurrent schedules, a 3-sec changeover delay (COD) was employed, and pecks on the dark center key had no programmed effect.

The remainder of the experiment consisted of exposure to various pairs of *conc* RI RI schedules. All animals began with 10 sessions

on *conc* RI 64-sec RI 64-sec ($T_{L,R}$ = 20 sec, $P_{L,R}$ = 20/64). Two groups were then differentiated in regard to the parameters, T and P, of the pairs of schedules. For Group T, the members of a concurrent pair differed in T but not in P; for Group P, the components differed in P but not in T. After the first 10 sessions, Group T advanced to 27 sessions on *conc* RI 128-sec RI 42.67-sec (T_L = 27 sec, T_R = 9 sec, $P_{L,R}$ = 27/128) and then to 13 sessions on *conc* RI 192-sec RI 38.40-sec (T_L = 45 sec, T_R = 9 sec, $P_{L,R}$ = 30/128). For Group P, 27 sessions on *conc* RI 128-sec RI 42.67-sec ($T_{L,R}$ = 20 sec, P_L = 20/128, P_R = 60/128) were followed by 13 sessions on *conc* RI 192-sec RI 38.40-sec ($T_{L,R}$ = 18 sec, P_L = 12/128, P_R = 60/128). Subscripts indicate left- and right-key schedule parameters. To reduce the possible systematic influence on the data of peculiarities of the individual chambers, the birds were placed in a different chamber each day according to a simple rotation scheme. Sessions lasted 1 hr, and the reinforcer was a 3-sec access to mixed grain.

Results and Discussion

Responses and obtained reinforcers for both alternatives were averaged over the last five sessions for each bird and each concurrent pair. The means are given in Table 1. The values in parentheses are the standard errors of the neighboring means. Inspection of these data indicates that the standard errors are quite small, given the value of the means. It can be assumed that the mean values are representative of the five sessions on which they are based.

To estimate values of k and a for each animal, the logarithms of the response and obtained reinforcer ratios derived from Table 1 were used. Parameters of the following equation were estimated by the method of least squares:

$$\log(B_1/B_2) = a\log(r_1/r_2) + \log k. \qquad (4)$$

This is simply Equation 3 in logarithmic form. The results are given in Table 2, column (a). In column (b), log$k$ was fixed at zero, and a was treated as a free, fitted, parameter. Column (c) gives the results obtained when a was fixed at 1.0 and log$k$ was free to vary. The values in parentheses are measures of goodness of fit. These are the variance of $\log(B_1/B_2)$ about the fitted line divided by the variance of log (B_1/B_2). The smaller the value the better the fit. Column (d) displays the fit when a = 1.0 and log$k$ = 0.0: this is the logarithmic form of Equation 2. Note that the table gives k, not

Table 1

Means and standard errors for last five sessions on each schedule for each animal.

	Responses/Minute		*Reinforcers/Hour*	
Animal	*Left*	*Right*	*Left*	*Right*
Group T				
7018	50.5 (1.5)	39.5 (3.0)	52.4 (3.7)	49.0 (3.1)
	30.6 (1.9)	85.0 (2.6)	27.6 (2.9)	74.8 (3.9)
	26.3 (2.5)	88.2 (6.8)	15.8 (1.2)	82.2 (3.4)
8236	35.4 (3.5)	26.6 (2.3)	47.6 (1.3)	41.6 (2.4)
	21.4 (1.7)	44.4 (2.9)	24.8 (0.5)	74.6 (2.9)
	16.8 (1.2)	45.5 (2.4)	15.6 (0.6)	79.0 (2.6)
7036	31.2 (2.8)	32.6 (4.4)	41.6 (3.7)	43.0 (4.7)
	15.8 (2.8)	44.1 (4.6)	19.4 (3.7)	70.6 (9.0)
	12.9 (1.5)	45.4 (3.4)	15.2 (0.9)	72.0 (3.9)
8271	30.5 (1.3)	36.2 (1.9)	48.0 (3.5)	50.8 (0.9)
	21.3 (0.7)	49.5 (2.4)	19.8 (1.2)	79.6 (3.0)
	15.3 (1.0)	57.5 (3.7)	13.8 (1.8)	89.8 (2.9)
8122	42.8 (1.9)	52.2 (1.7)	47.8 (2.6)	49.2 (3.0)
	31.0 (3.5)	71.1 (4.1)	23.8 (1.4)	72.8 (2.6)
	18.0 (2.0)	74.8 (4.1)	15.2 (1.0)	87.2 (4.6)
8062	52.8 (3.2)	45.4 (2.2)	46.2 (1.2)	43.6 (3.2)
	10.3 (0.6)	71.7 (6.4)	16.0 (0.9)	70.8 (2.1)
	18.4 (1.4)	101.6 (7.6)	15.2 (1.6)	80.2 (3.9)
7804	42.9 (2.1)	41.8 (6.0)	45.0 (1.4)	44.6 (3.0)
	13.1 (0.9)	82.5 (2.3)	22.8 (2.4)	79.2 (2.3)
	11.3 (1.7)	68.3 (2.8)	13.8 (1.7)	75.2 (2.6)
7024	23.9 (1.9)	28.3 (1.8)	40.6 (3.0)	46.8 (2.3)
	10.3 (0.8)	55.1 (5.0)	20.4 (0.5)	66.6 (3.0)
	8.7 (1.4)	86.6 (6.7)	12.8 (1.1)	83.4 (1.5)
Group P				
7063	52.3 (4.1)	42.7 (3.7)	48.6 (2.6)	43.4 (3.4)
	30.8 (2.2)	55.2 (4.0)	21.6 (1.6)	77.0 (1.7)
	30.6 (1.9)	53.4 (2.3)	19.0 (2.1)	80.4 (2.2)
7760	43.5 (5.2)	34.6 (3.8)	37.8 (6.2)	41.8 (2.3)
	27.3 (1.8)	101.6 (4.8)	21.0 (2.3)	72.8 (3.7)
	16.2 (2.3)	101.6 (6.9)	14.2 (1.5)	90.8 (3.0)
8306	27.6 (2.6)	23.9 (2.0)	45.4 (3.1)	43.6 (4.0)
	11.9 (2.0)	37.8 (6.0)	20.0 (3.6)	66.4 (7.4)
	12.4 (1.2)	53.2 (2.2)	15.6 (2.4)	84.8 (2.6)
7582	24.1 (1.5)	27.2 (3.6)	43.8 (2.5)	44.4 (3.0)
	15.0 (2.0)	41.2 (4.3)	21.4 (1.9)	71.6 (0.7)
	12.7 (1.3)	56.7 (4.4)	12.8 (1.0)	82.8 (2.2)
7722	49.4 (8.4)	52.5 (1.9)	44.8 (3.9)	45.8 (2.8)
	23.4 (1.7)	88.6 (6.5)	24.4 (1.6)	79.0 (1.3)
	12.1 (1.9)	87.3 (6.5)	11.2 (1.5)	91.8 (3.3)
7635	27.9 (2.9)	30.6 (3.5)	41.0 (3.6)	48.2 (2.1)
	10.9 (1.5)	39.6 (2.8)	18.0 (2.9)	72.8 (2.5)
	14.7 (1.4)	59.2 (3.4)	14.2 (1.6)	74.6 (3.7)
7595	28.2 (1.6)	32.4 (2.2)	44.8 (1.0)	45.0 (2.4)
	15.4 (0.8)	50.4 (2.2)	19.6 (1.4)	75.8 (3.2)
	15.6 (2.2)	48.0 (1.7)	14.0 (1.5)	88.6 (2.7)

Table 2

Values of k and a and Goodness-of-fit ( )

	(a)		(b)	(c)	a=1 (d)
	k =	a =	k=1 a=	a=1 k=	k=1
GROUP T					
7018	0.90	0.89 (0.07)	0.82 (0.08)	0.82 (0.08)	(0.18)
8236	0.85	0.74 (0.01)	0.64 (0.05)	0.68 (0.12)	(0.61)
7036	1.01	0.79 (0.00)	0.80 (0.01)	0.83 (0.07)	(0.19)
8271	1.12	0.61 (0.03)	0.67 (0.05)	0.72 (0.44)	(0.92)
8122	1.16	0.70 (0.02)	0.80 (0.05)	0.86 (0.20)	(0.29)
8062	0.95	1.18 (0.04)	1.15 (0.04)	1.15 (0.06)	(0.08)
7804	1.07	1.16 (0.07)	1.21 (0.07)	1.25 (0.09)	(0.16)
7024	1.06	1.24 (0.01)	1.28 (0.02)	1.37 (0.05)	(0.18)
GROUP P					
7063	0.88	0.52 (0.02)	0.43 (0.07)	0.58 (0.88)	(3.19)
7760	0.74	1.20 (0.01)	1.00 (0.05)	0.91 (0.04)	(0.05)
8306	0.93	0.94 (0.01)	0.89 (0.01)	0.88 (0.01)	(0.05)
7582	1.12	0.74 (0.00)	0.81 (0.01)	0.86 (0.12)	(0.20)
7722	1.11	0.93 (0.02)	0.99 (0.02)	1.02 (0.02)	(0.02)
7635	0.96	0.90 (0.01)	0.87 (0.01)	0.86 (0.02)	(0.08)
7595	1.21	0.58 (0.09)	0.70 (0.15)	0.78 (0.55)	(0.82)
GROUPED DATA					
Group T	1.00	0.93 (0.19)	0.93 (0.19)	0.93 (0.19)	(0.20)
Group P	0.97	0.85 (0.14)	0.83 (0.14)	0.83 (0.16)	(0.24)

log$k$, for convenient reference to Equation 3. Inspection of Table 2, column (a), shows Equation 3 to be an excellent representation of the data: the goodness-of-fit measures are all less than 0.10. The values in column (d) indicate that Equation 2 is inadequate in 10 of the 15 cases if the criterion for fit of 0.10 is accepted. Data for Subject #7063 are a clear violation of Equation 2. Note, however, that for 14 of the 15 birds, #7595 is the exception, column (b) conditions produce excellent fit.

Comparison of the two groups suggests that Group T may tend to have higher values of a, in column (a), than does Group P. The grouped data at the bottom of Table 2 obtained by pooling data for all animals in a group and finding a single regression line support this conclusion. However, the values of a for individual subjects show great overlap of the two groups.

The curve-fitting analysis of data for individual subjects indicates that Equation 3 was supported, but it can be reduced to an equation with only one free parameter, a. Further, the method of generating the RI schedules, the differentia of the two groups, had little if any effect on the value of a. Statistical procedures for parameter estimation from sample data should confirm these conclusions and provide values of k and a.

Statistical procedures, using the t distribution, for setting the 90% confidence intervals about the means of log$k$ and log$a$ were applied to the logarithms of the values in Table 2, column (a). The antilogs of the results were as follows. For Group T, k was found to be 1.01, with a 90% confidence interval of 0.94 to 1.08; the values of a were 0.89 and 0.74 to 1.06. Corresponding values for Group P were: k = 0.98, 0.86 to 1.11, and a = 0.80, 0.64 to 1.01. When data for both groups were pooled, k was found to be 1.00, with an interval of 0.94 to 1.06, and a was 0.84 with an interval of 0.74 to 0.96. The value of k clearly tended toward 1.0, and the value of a tended to be less than 1.0. Note the greater variability in a. These results confirm the conclusions of the previous analysis and suggest that the parameter a is less than 1.0, the common finding of "undermatching" (Baum, 1974).

Given these estimates of k and a, it is possible to make a quantitative comparison of the results of *conc* RI RI and *conc* VI VI schedules. de Villiers (1977, p. 240) presented data similar to those of Table 2, column (a), for 23 birds from seven laboratories employing *conc* VI VI schedules. The present author applied the statistical analysis given above to these data. The value of k was 0.96, with a 90% confidence interval of 0.90 to 1.02; corresponding values for a were 0.82 and 0.76 to 0.89. The overlap of parameter values for *conc* RI RI and *conc* VI VI schedules is impressive. When the results of the present experiment were pooled with those from de Villiers, n = 38, the value of k was 0.97, with a 90% confidence interval of 0.93 to 1.02. Corresponding values for a were 0.83 and 0.78 to 0.89. A t test of log$k$ = 0.0 produced t (37) = −1.009, and the test of log$a$ = 0.0 produced t (37) = −4.609. For a two-tailed test, there is no evidence against log$k$ = 0.0 ($p > 0.30$), but there is strong evidence against log$a$ = 0.0 ($p < 0.01$). Based on accumulated data for *conc* RI RI and *conc* VI VI schedules it is concluded that k in Equation 3 is 1.0 and a is less than 1.0, approximately 0.83.

EXPERIMENT II

It is known that Equation 2 does not account for data from *conc* FI VI schedules (Baum, 1974). The RI schedules fall on a continuum between FI and VI schedules. When P

is 1.0, one has an FI, and as P moves from 1.0, VI-like schedules are produced. How FI-like can one component of a *conc* RI RI schedule be without violating the prediction of Equation 2? This experiment was designed to demonstrate the effects of long intervals, T, and large probabilities, P, in regard to Equation 2.

METHOD

Subjects

Six experimentally naive White King pigeons were maintained at 80% to 85% of their free-feeding weights throughout the experiment.

Apparatus

The apparatus was the same as in Experiment I.

Procedure

Preliminary training was the same as in Experiment I. Fifteen 1-hr sessions followed on Schedule 1, *conc* RI 64-sec RI 64-sec ($T_L = 1$ sec, $P_L = 2/128$, $T_R = 43$ sec, $P_R = 86/128$). Fifteen more sessions followed on Schedule 2, *conc* RI 64-sec RI 64-sec ($T_L = 1$ sec, $P_L = 2/128$, $T_R = 57$ sec, $P_R = 114/128$). For both concurrent pairs, the mean interreinforcement times on the two alternatives were equal, but the parameters, T and P, were quite different.

RESULTS AND DISCUSSION

Responses and obtained reinforcers were averaged over the last five sessions on each schedule for each bird. The means and standard errors are shown in Table 3. As in Experiment I, the means appear to be representative of the sessions entering into their determination. Since both schedules provided essentially the same ratios of reinforcer densities, approximately 1.0, curve fitting for Equation 3 is uninformative: the correlation of log (B_1/B_2) with $\log(r_1/r_2)$ is expected to be near zero, and this implies a value for a of zero. Further, the value of k might be expected to change systematically as the schedule parameters change (Baum, 1974), and this suggests an evaluation of k for each schedule. Thus, a different implication of Equation 3 was examined. Rearrangement of the logarithmic form of Equation 3 with $a = 1$ produces:

$$\log(B_1/B_2) - \log(r_1/r_2) = \log k. \qquad (5)$$

Table 3

Means and standard errors for last five sessions on each schedule for each animal.

	Responses/Minute		*Reinforcers/Hour*	
Animal	*Left*	*Right*	*Left*	*Right*
7228	29.7 (2.1)	29.5 (2.5)	47.8 (2.4)	40.0 (1.9)
	35.0 (5.9)	30.7 (2.3)	48.6 (3.6)	48.0 (1.4)
8098	41.4 (1.8)	41.1 (3.2)	44.8 (3.0)	48.2 (2.1)
	49.7 (1.9)	57.0 (3.7)	47.0 (2.2)	47.8 (1.0)
8137	53.2 (1.8)	37.4 (2.6)	46.8 (1.5)	46.4 (1.4)
	63.4 (4.4)	36.3 (2.6)	48.2 (3.0)	46.6 (1.2)
8114	49.6 (2.8)	34.8 (2.6)	54.8 (2.9)	43.0 (1.8)
	60.7 (3.1)	32.3 (3.3)	50.0 (1.1)	43.8 (1.4)
7149	31.7 (3.2)	32.3 (2.1)	39.2 (3.4)	51.2 (0.7)
	42.1 (1.8)	37.2 (2.8)	49.6 (3.7)	48.0 (1.8)
8131	42.6 (3.1)	38.6 (2.6)	48.4 (3.6)	43.8 (2.4)
	60.0 (6.5)	34.7 (3.7)	42.6 (4.0)	44.4 (2.1)

If logk in Equation 5 is zero, then Equation 2 is supported. However, $\log k \neq 0$ argues for retention of the parameter k in Equation 3. Baum's (1974) analysis of data from Trevett, Davison, and Williams (1972) for *conc* FI VI schedules indicated preference for the VI alternative. Therefore, when the ratios in Equation 5 are formed as VI-like/FI-like alternatives, the left-key relative to the right-key alternatives of the present experiment, logk greater than zero is expected. The statistical hypothesis implied by Equation 2 can be tested as the "null" hypothesis usually is, and the data from *conc* FI VI schedules specify the alternative to this "null" hypothesis.

For each schedule, pooled data for the six subjects, derived from Table 3 for use in Equation 5, were evaluated in terms of the t distribution. For Schedule 1, t (5) = 1.355 for the mean difference of 0.0435. For Schedule 2, t (5) = 2.402 for the mean difference of 0.1136. For Schedule 2, but not for Schedule 1, there was evidence against $t = 0$ at the 0.05 level, one-tailed. As *conc* RI RI schedules converge on *conc* FI RI schedules, the results in terms of Equation 3 become similar to those obtained with *conc* FI VI schedules.

GENERAL DISCUSSION

Random-interval schedules establish a well-defined continuum bounded by the FI and VI schedules. As T approaches T/P, the RI schedules approach the FI; with P much less than 1.0, the VI schedules are approximated.

Farmer (1963) showed that the length of the postreinforcement pause in RI schedules is a joint function of T and T/P. It is reasonable to assume that *conc* RI RI schedules will produce data sometimes similar to concurrent FI and sometimes similar to concurrent VI situations. The results of Experiment I were very close to those from *conc* VI VI experiments; results of Experiment II were similar to ones from *conc* FI VI schedules. In both cases, Equation 3 was supported and Equation 2 was not. Baum's (1974) demonstration that qualitatively different schedules, *conc* FI VI, cause $k \neq 1.0$ was confirmed in Experiment II. His generalization that undermatching, $a < 1$, is the common result in *conc* VI VI situations was confirmed in Experiment I.

The results of Experiment II suggest that any variable that produces failure of Equation 2 under the condition of equal reinforcer densities will result in $k \neq 1.0$ in Equation 3. This follows, since that concurrent pair sets the origin for Equation 3, or at least strongly influences the value resulting from curve fitting. In Experiment I, the component schedules were identical, in regard to parameters, at the point for equal reinforcer densities, and $k = 1.0$ resulted. The same condition probably obtained in the experiments summarized by de Villiers (1977); $k = 1.0$ for those data.

Why is $a < 1.0$ in Equation 3? The parameter a is estimated by the least-squares procedure as the covariance of $\log(B_1/B_2)$ with $\log(r_1/r_2)$ divided by the variance of the latter. Therefore, any variable that tends to attenuate the covariance results in $a < 1.0$. At this time, the list of candidates must be long, and it probably includes most of the variables of general interest in the field of operant conditioning. Perhaps the best research tactic is to manipulate variables, with the aim of finding a set that strongly influences the value of a. These variables may then be added to Equation 2, and a more general empirical relation may be established. The relation will require examination of composition rules for the variables. To search for some condition that produces $a = 1.0$, given the comment above, is unlikely to be fruitful.

Herrnstein (1970) and de Villiers (1977) have been interested in a version of the matching law, which entails addition of parameters. The parameters carry more theoretical weight than those of Equation 3. The effort suggests the possibility of a rational law built on an empirical law and should, therefore, be examined carefully. For concurrent schedules of the kind from Experiment I, the "new" law may be written as:

$$\frac{B_1}{B_2} = \frac{r_1}{r_2}\left[\frac{r_2 + mr_1 + r_0}{r_1 + mr_2 + r_0}\right]. \qquad (6)$$

The value of m is assumed to be 1.0 for concurrent schedules. The term r_0 is an extra source of reinforcement. If $m = 1.0$, then Equation 6 reduces to the, false, Equation 2. On the other hand, if $B_1/B_2 = (r_1/r_2)^a$, Experiment I, then for $r_1 > r_2$, the bracketed quantity must be less than 1.0. Given the inequality, it is easily shown that $m < 1.0$ in Equation 6. The rationale for setting $m = 1.0$ should be reconsidered.

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